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$3a^2(m-\frac{1}{m})+4ay+(x^2+y^2-4ax)(m-\frac{1}{m})=0$; or

$$3a^2p+4ay+(x^2+y^2-4ax)p=0\ldots(\text{II}).$$

From (II), we get $p=-\frac{4ay}{x^2+y^2-4ax+3a^2}$, and substituting this in (I), we get

$$(x^2+y^2+3a^2-4ax)^2(3a^2+10ax-x^2-y^2)-16a^2y^2(x^2+y^2-4ax)=0.$$

Also solved by M. A. Harding, S. Lefschetz, and A. H. Holmes.

367. **Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Michigan.**

Solve the simultaneous equations:

$$\frac{2x}{1+x^2}=y\ldots(1);\quad \frac{2y}{1+y^2}=z\ldots(2);\quad \frac{2z}{1+z^2}=u\ldots(3);\quad \frac{2u}{1+u^2}=x\ldots(4).$$

Solution by PROFESSOR F. L. GRIFFIN, Reed College, Portland, Oregon.

By inspection three solutions are $x=y=z=u=1$, or -1, or 0; and there can be but fourteen others. Now let $x=i\tan\theta$, $[i=\sqrt{-1}]$, whence $y=i\tan 2\theta$, $z=i\tan 4\theta$, $u=i\tan 8\theta$, and $x=i\tan 16\theta$. But $\tan 16\theta=\tan\theta$ for finite values of θ only if $16\theta=\theta+n\pi$, or $\theta=n\pi/15$. Thus we have

$$\begin{array}{rlllll}
x=0, & i\tan\pi/15, & i\tan 2\pi/15, & i\tan 3\pi/15, & \ldots, & i\tan 14\pi/15;\\
y=0, & i\tan 2\pi/15, & i\tan 4\pi/15, & i\tan 6\pi/15, & \ldots, & i\tan 28\pi/15;\\
z=0, & i\tan 4\pi/15, & i\tan 8\pi/15, & i\tan 12\pi/15, & \ldots, & i\tan 56\pi/15;\\
u=0, & i\tan 8\pi/15, & i\tan 16\pi/15, & . \quad . \quad . & . \quad . & . \\
\end{array}$$

the values for y, z, u being of course the same sets as for x in different orders. The values $x=+1$, -1 correspond to infinite values of θ for which $\tan\theta=-i$, $+i$.

GEOMETRY.

393. **Proposed by S. LEFSEHETZ, University of Nebraska.**

Draw a triangle having a given angle, and with its vertices on three given concentric circles.

I. Solution by H. C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Nebraska.

Let $x^2+y^2=c^2$, $x^2+y^2=b^2$, $x^2+y^2=a^2$, be the given circles, and ϕ the given angle. Place the vertex of the given angle, ϕ, on the circumference of the first circle at 0, c, the angle being formed by the lines,

$$y=lx+c, \text{ and } y=\frac{l+\tan\phi}{1-l\tan\phi}x+c,$$

where l is the tangent of the angle made by the first line with the x-axis, in the usual sense.

Substituting the equation of the first line in the equation of the second circle, the condition for intersection is

$$l\geqq\frac{\sqrt{(c^2-b^2)}}{b}, \text{ or } l\leqq-\frac{\sqrt{(c^2-b^2)}}{b},$$

and substituting the equation of the second line in the equation of the third circle, the condition for intersection is,

$$\frac{l+\tan\phi}{1-l\tan\phi}\geqq\frac{\sqrt{(c^2-a^2)}}{a}, \text{ or } \frac{l+\tan\phi}{1-l\tan\phi}\leqq-\frac{\sqrt{(c^2-a^2)}}{a},$$

which reduces to

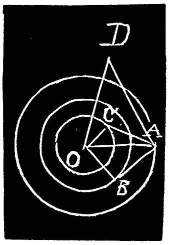
$$l\geqq\frac{a\tan\phi-\sqrt{(c^2-a^2)}}{\sqrt{(c^2-a^2)}\tan\phi+a}, \text{ or } l\leqq\frac{a\tan\phi+\sqrt{(c^2-a^2)}}{a-\sqrt{(c^2-a^2)}\tan\phi}.$$

Taking $c>b$ and $c>a$, there is no solution if

$$\phi>\tan^{-1}\frac{b\sqrt{(c^2-a^2)}+a\sqrt{(c^2-b^2)}}{\sqrt{(c^2-a^2)}\sqrt{(c^2-b^2)}-ab},$$

with the vertex of ϕ on the circumference of the first circle. But any angle gives a solution by placing its vertex on the circumference of either of the other circles, by solving in a similar manner. With these exceptions, any value of l, except the limiting value, which gives only one solution, gives four solutions.

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.**



With one angle given, there is an indefinite number of triangles, and consequently I will modify the problem to having the three angles given; or, in other words, to describe a triangle whose vertices are in the circumference of three given concentric circles similar to a given triangle.

On the radius OA draw the triangle OAD similar to the given one; draw DC from $OA:OB=AD:DC$. Make $\angle AOB=\angle ADC$, connect B with C, then ABC will be the required triangle. For, by construction, $\triangle ABC$ is similar to $\triangle AOB$, and, consequently, $\triangle ABC$, the vertices of which lie on the three circumferences, is similar to $\triangle OAD$.

Solved similarly by C. N. Schmall.

394. Proposed by W. J. GREENSTREET, M. A., Editor, The Mathematical Gazette, Stroud, England.

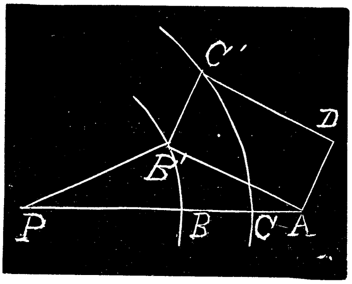
The joins of the excentres to the corresponding vertices of the pedal triangle are concurrent.

No solution of this problem has been received.

395. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

From a point P without a rectangular field ABC the distances PA, PB, PC measured to the corners are respectively 70, 40, 60 chains. What is the area of the field?

Solution by A. H. HOLMES, Brunswick, Maine; J. SCHEFFER, A. M., Hagerstown, Maryland, and M. A. MUZZY, Waterville, Maine.



Let P be the point without the field. Draw the line PBA; on it lay off $PB=40$ chains, $PC=60$ chains, and $PA=70$ chains. Then any point on the circumference of circle whose center is P and radius PB may be one corner of the rectangular field. One such field is $AB'C'D$. Hence, the problem is indeterminate. To make the problem determinate, one of the angles APB', APC', or $B'PC'$ should be given.

396. Proposed by DANIEL KRETH, Oxford, Iowa.

In the triangle ABC, $AB=214$, $BC=263$, and $AC=405$. A point P is situated in the same horizontal plane; angle $BPA=13^\circ\ 30$ and angle $BPC=29^\circ\ 50'$. Find the distances, AP, BP, and CP.

Solution by M. A. MUZZY, Waterville, Maine.

Draw a circle through P, A, C, cutting BP in some point D. Then $BPA=ACD$, $BPC=CAD$, or $BPC=180^\circ-CAD$. Hence $CAD=29^\circ\ 50'$ or $150^\circ\ 10'$, $ADC=136^\circ\ 40'$ or $16^\circ\ 20$. There are four solutions.

In triangles CAD find $CD_1=CD_2=293.60$, $CD_3=CD_4=716.43$.

In triangle ABC find $ACB=28^\circ\ 23.8'$. $BCD=ACB\mp ACD$. Whence $BCD_1=BCD_4=14^\circ\ 53.8'$, $BCD_2=BCD_3=41^\circ\ 53.8'$.

In triangles BCD find $CBD_1=105^\circ\ 21.2'$, $CBD_2=77^\circ\ 13.4'$, $CBD_3=119^\circ\ 27.8'$, $CBD_4=156^\circ\ 46.0'$.

In triangles CBP find $BP_1=372.59$, $BP_2=389.09$, $BP_3=269.93$, $BP_4=422.58$; $CP_1=509.80$, $CP_2=515.58$, $CP_3=460.29$, $CP_4=208.55$.